# Nuclear Physics Summer School
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# Some Results-1

A new exact symmetry for baryons as $N_c \to \infty$

- A $SU(6)_c$ spin-flavor symmetry that connects the six states $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$, $s\uparrow$, $s\downarrow$.
- Baryons form an irrreducible representation of the spin-flavor algebra.
- Relates octet $N, \Lambda, \Sigma, \Xi$ and decuplet $\Delta, \Sigma^*, \Xi^*, \Omega$.

# Some Results-2

Can compute $1/N_c$ corrections in a systematic expansion, and the expansion is useful for $N_c = 3$

- $1/N_c$ corrections can be classified by their spin-flavor transformation properties
- Relations obtained to various orders in $1/N_c$. $1/N_c = 1/3$ factors evident in the experimental data.
- $1/N_c$ corrections comparable in size to $SU(3)$ breaking corrections due to $m_s$

# Some Results-3

- Provides a deeper understanding of the success of quark models.
- Many results obtained in the nonrelativistic quark model, bag model, or Skyrme model, can be proven in QCD to order $1/N_c$ or $1/N_c^2$.
- $SU(6)_c$ is the underlying symmetry that relates quark models to each other and to QCD.

# Some Results-4

Tells you how to consistently apply chiral perturbation theory to baryons

- $N_c$ and $\Delta$ states have to be treated together
- Cancellations in chiral loops
- Form of $SU(3)$ symmetry breaking due to $m_s$ is constrained by the $1/N_c$ expansion.
- Provides new insights into $SU(3)$ breaking.

# Some Results-5

New predictions for heavy baryon properties

- Relates heavy quark baryons to the nucleon
- Compute masses and pion couplings of the $\Lambda_c$, $\Lambda_b$, etc Results are in good agreement with experiment.
- Can combine $1/N_c$ and $1/m_Q$ expansions

# Some Results-6

New predictions for excited baryons
(Carlson, Carone, Goity, Schat, Lebed, Pirjol, Yan)

Nucleon Potential

- Explains the size of terms in the nucleon potential
- Gives Wigner supermultiplet symmetry in light nuclei

# Large-$N_c$ Baryons

$$\epsilon_{i_1 i_2 i_3 \cdots i_{N_c}}\ q^{i_1} q^{i_2} q^{i_3} \cdots q^{i_{N_c}}$$

bound state of $N_c$ quarks
completely antisymmetric in color

completely symmetric in the quarks, since Fermi-statistics compensated by color antisymmetry.

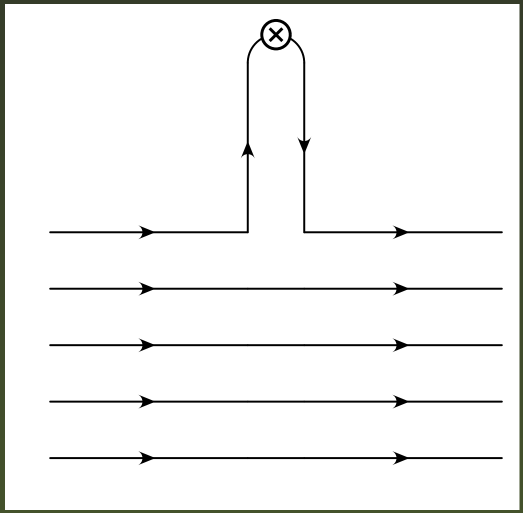
# $N_c$ Counting Rules for Baryons

Baryon is made of $N_c$ quarks.

- Baryon mass is order $N_c$
- Baryon size is order $\Lambda_{\mathrm{QCD}}^{-1}$ (order one)
- Baryon-meson coupling is $\leq \sqrt{N_c}$
- Each extra meson costs $1/\sqrt{N_c}$
- One-body matrix element $\langle B|\,\bar{q}\,\Gamma\,q\,|B\rangle \leq N_c$
- Two-body matrix element $(\bar{q}\,\Gamma\,q\,\bar{q}\,\Gamma\,q) \leq N_c^2$

# Baryon-Meson Couplings

baryon-meson vertex $\sim O\left(\sqrt{N_c}\right)$



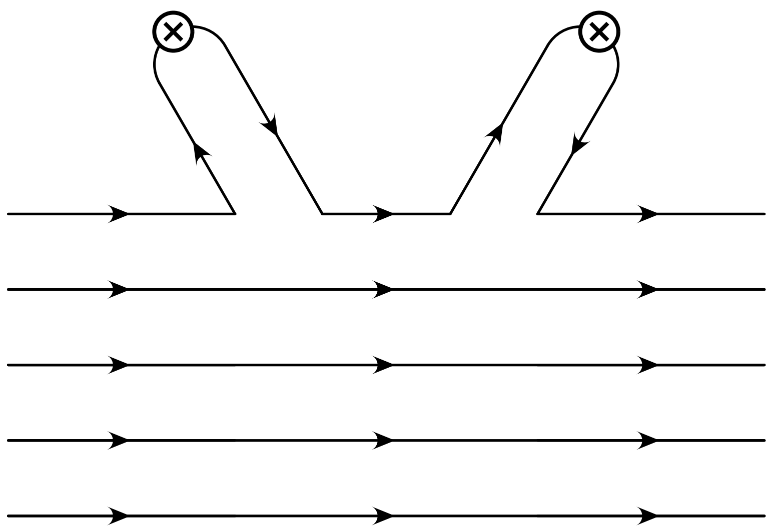
$$N_c\left(\frac{1}{\sqrt{N_c}}\right)$$

$\otimes = \bar{q}q/\sqrt{N_c}$ creates a meson with unit amplitude

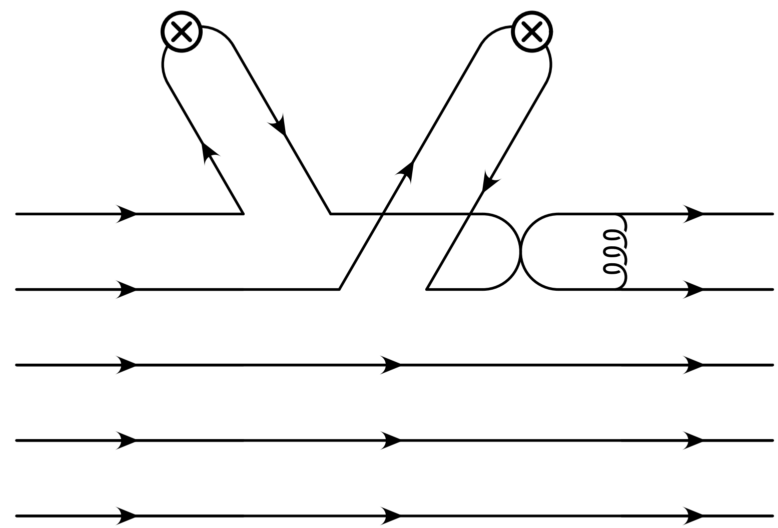
# Baryon-Meson Scattering

baryon + meson $\rightarrow$ baryon + meson $\sim O(1)$

$$N_c \left(\frac{1}{\sqrt{N_c}}\right)^2 \qquad\qquad N_c^2 \left(\frac{1}{\sqrt{N_c}}\right)^2 \left(\frac{1}{N_c}\right)$$



(a)



(b)

# Baryon-Pion Scattering

- $M_{\mathrm{baryon}} \sim O(N_c)$, so baryon acts as heavy static source
- Baryon propagator

$$\frac{i\,(\not{P} + M)}{P^2 - M^2} \to \frac{i}{k \cdot v}\left(\frac{1 + \not{v}}{2}\right) \to \frac{i}{E}$$

[Not only for pions. Argument is cleanest in this case.]

- $BB'\pi$ vertex $\sim O(\sqrt{N_c})$

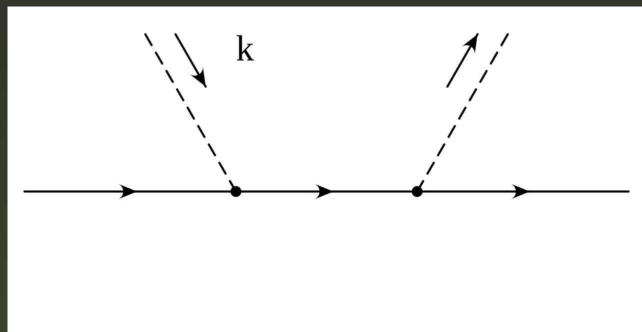
$$\frac{\partial_\mu \pi^a}{f_\pi} \left(A^{\mu a}\right)_{B'B}$$

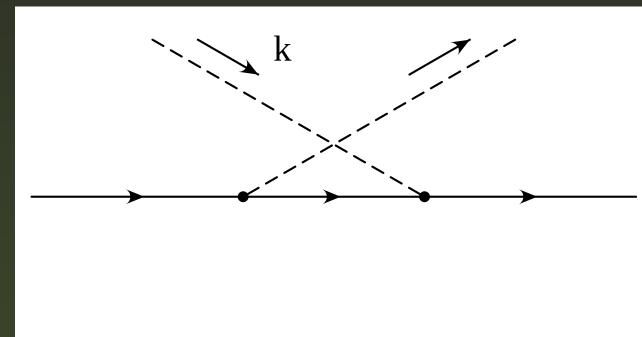
$$\left(A^{\mu a}\right)_{B'B} = \langle B' | \bar{q} \gamma^\mu \gamma_5 \tau^a q | B \rangle \sim O(N_c)$$

- $N_c \to \infty$ limit

$$\frac{\partial^i \pi^a}{f_\pi} \left(A^{ia}\right)_{B'B}$$

$$A^{ia} \equiv g N_c X^{ia}$$




$$N_c \left[ X^{ia}, X^{jb} \right] \leq O(1)$$

$$X^{ia} = X_0^{ia} + \frac{1}{N_c} X_1^{ia} + \frac{1}{N_c^2} X_2^{ia} + \ldots$$

$$\left[ X_0^{ia}, X_0^{jb} \right] = 0$$

# Spin-Flavor Symmetry

- Consistency conditions for scattering of low-energy pions with baryons at large-$N_c$ leads to derivation of contracted spin-flavor symmetry for baryons
- Consistency of large-$N_c$ power counting rules for baryon-meson scattering amplitudes and vertices leads to non-trivial constraints on $1/N_c$ corrections to large-$N_c$ baryon matrix elements

# Contracted Spin-Flavor Algebra

$$\left[J^i, I^a\right] = 0,$$

$$[J^i, J^j] = i\epsilon^{ijk} J^k, \qquad \left[I^a, I^b\right] = i\epsilon^{abc} I^c,$$

$$\left[J^i, X_0^{ja}\right] = i\epsilon^{ijk} X_0^{ka}, \quad \left[I^a, X_0^{ib}\right] = i\epsilon^{abc} X_0^{ic},$$

$$\boxed{\left[X_0^{ia}, X_0^{jb}\right] = 0}$$

# Induced Representations

So the starting point is to work out the irreducible representations of the contracted symmetry, and then classify the $1/N_c$ corrections.

Standard theory of induced representations (e.g. Mackey) gives the Skyrme model.

Infinite dimensional unitary representations

# Large-$N_c$ Skyrme Model

- Simple understanding of spin-flavor generator $X_0^{ia}$ as collective coordinate

$$X_0^{ia} = \mathrm{tr}\, A\tau^i A^{-1}\tau^a$$

- Contracted spin-flavor symmetry for baryons in $N_c \to \infty$ limit realized exactly since

$$\left[X_0^{ia}, X_0^{bj}\right] = 0.$$

# Quark Model

The SU(6) generators are

$$\begin{aligned} J^i &= q^\dagger \, \frac{\sigma^i}{2} \, q \\ T^a &= q^\dagger \, \frac{\tau^a}{2} \, q \\ G^{ia} &= q^\dagger \, \frac{\sigma^i}{2} \frac{\tau^a}{2} \, q \end{aligned}$$

with $\left[q, q^\dagger\right] = 1$.

This gives

$$
\begin{aligned}
\left[J^i, G^{jb}\right] &= i\epsilon^{ijk}\ G^{kb} \\
\left[I^a, G^{jb}\right] &= i\epsilon^{abc}\ G^{jb} \\
\left[G^{ia}, G^{jb}\right] &= \frac{i}{4}\epsilon^{ijk}\delta^{ab}\ J^k + \frac{i}{4}\delta^{ij}\epsilon^{abc}\ T^c
\end{aligned}
$$

Let

$$G^{ia} = N_c X^{ia},$$

and take the limit $N_c \to \infty$. This reduces to the QCD symmetry derived earlier.

# $N_F = 2$

$$J = I = \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \cdots,\ \frac{N_c}{2}$$

$$N, \Delta, \ldots$$

An infinite tower of degenerate states as $N_c \to \infty$.

Known that the Skyrme and Quark models were equivalent as $N_c \to \infty$. By explicit calculation, and by a trick.

# $J = \frac{1}{2}$ $SU(3)_F$ rep

$$
\begin{array}{ccccccccccccccccccc}
 & & & & & & & & 1 & & 1 & & & & & & & & \\
 & & & & & & & 1 & & 2 & & 1 & & & & & & & \\
 & & & & & & 1 & & 2 & & 2 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 2 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 2 & & 2 & & 2 & & 1 & & & & \\
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 & & 1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 & & \\
 & 1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 & \\
1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 \\
 & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 &
\end{array}
$$

$\longleftarrow \frac{1}{2}(N_c+1)$ weights $\longrightarrow$

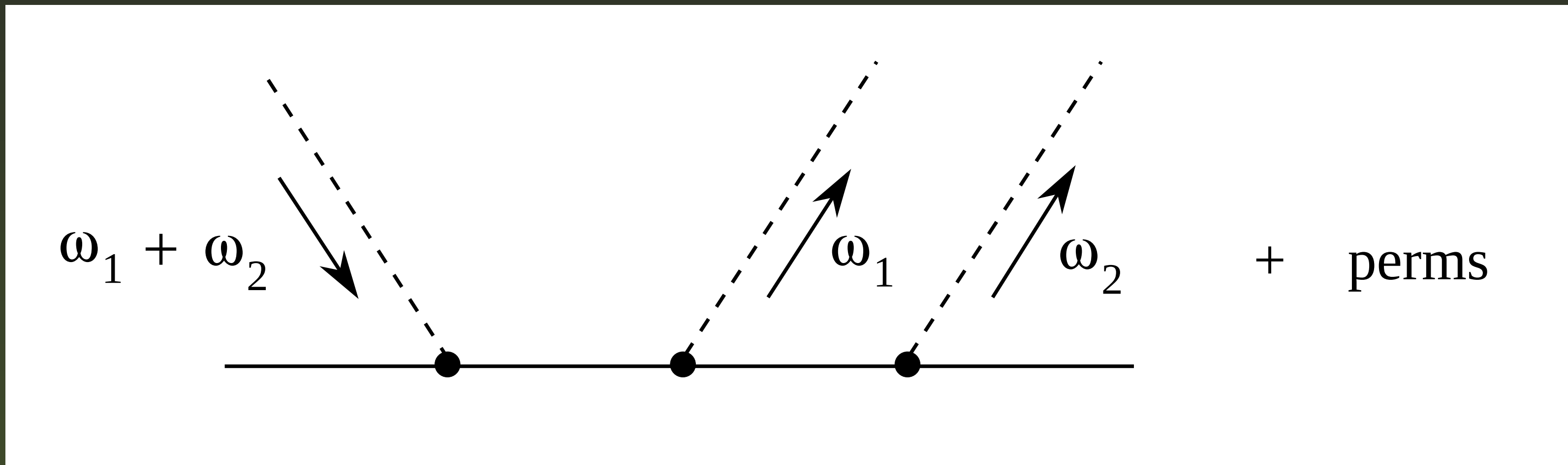
$J = \frac{3}{2}$

# $SU(3)_F$ **rep**

$$\begin{array}{ccccccccccccccccccc}
 & & & & & & 1 & & 1 & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 2 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 3 & & 3 & & 2 & & 1 & & & & \\
 & & & 1 & & 2 & & 3 & & 4 & & 3 & & 2 & & 1 & & & \\
 & & 1 & & 2 & & 3 & & 4 & & 4 & & 3 & & 2 & & 1 & & \\
 & 1 & & 2 & & 3 & & 4 & & 4 & & 4 & & 3 & & 2 & & 1 & \\
1 & & 2 & & 3 & & 4 & & 4 & & 4 & & 4 & & 3 & & 2 & & 1 \\
 & 1 & & 2 & & 3 & & 3 & & 3 & & 3 & & 3 & & 2 & & 1 & \\
 & & 1 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 1 & & \\
 & & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & 1 & & &
\end{array}$$

$$\longleftarrow \tfrac{1}{2}\left(N_c - 1\right) \longrightarrow$$

# $1/N_c$ CORRECTIONS




$$\text{Graph} \quad \propto \quad N_c^{3/2}\left[X^{ia},\left[X^{jb},X^{kc}\right]\right] + N_c^{3/2}\left[X^{ia},\left[X^{jb},\left[X^{kc},\Delta M\right]\right]\right]$$

Feynman diagrams give multiple commutators

Baryon propagator

$$\frac{i}{E - \Delta M} \to \frac{i}{E}\left(1 + \frac{\Delta M}{E} + \cdots\right)$$

Expand the vertex in $1/N_c$

$$X = X_0 + \frac{1}{N_c} X_1 + \frac{1}{N_c^2} X_2 + \ldots$$

Find

$$\left[X_0^{ia}, \left[X_0^{jb}, X_1^{kc}\right]\right] + \left[X_0^{ia}, \left[X_1^{jb}, X_0^{kc}\right]\right] \;=\; 0$$

$$\left[X_0^{ia}, \left[X_0^{jb}, \left[X_0^{kc}, \Delta M\right]\right]\right] \;=\; 0$$

# Results (two flavors)

$$\begin{aligned} X_1 &\propto X_0 \\ \Delta M &\propto \frac{J^2}{N_c} \end{aligned}$$

$$X = X_0 + \frac{1}{N_c} X_1 + \frac{1}{N_c^2} X_2 + \ldots$$

No $1/N_c$ corrections to ratio of pion couplings such as $g_{\pi NN}/g_{\pi N\Delta}$.

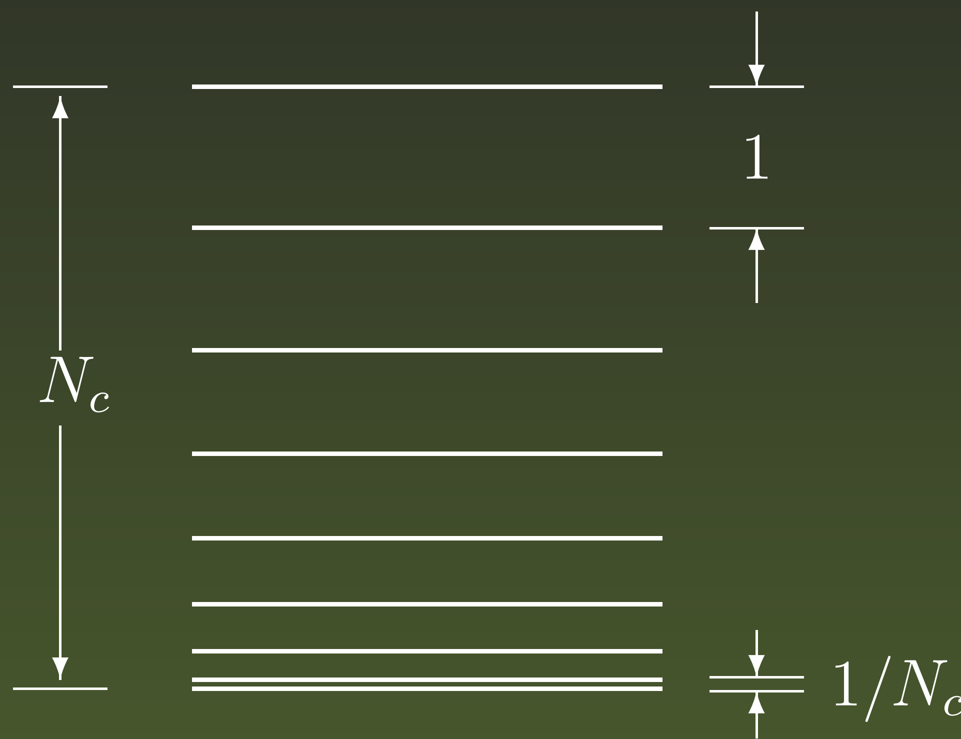
# Pion Couplings

Explains why the Skyrme/Quark model predictions work well for the ratios, but not for the absolute values. The ratios are the same as in QCD to order $1/N_c^2$.

| | Theory | Experiment |
|---|---|---|
| $g_{\pi N \Delta}$ | $13.2$ | $20.3$ |
| $g_{\pi NN}$ | $8.9$ | $13.5$ |
| $g_{\pi N \Delta}/g_{\pi NN}$ | $1.48$ | $1.5$ |

(From Adkins, Nappi, Witten)

# Hyperfine Mass Splittings




The $1/N_c$ corrections are small only in a part of the irreducible representation.

# Form of $1/N_c$ Expansion ($N_F = 2$)

$$N_c^{?}\ \mathcal{P}\left(X_0^{ia}, \frac{J^i}{N_c}, \frac{I^a}{N_c}\right)$$

Or one can use

$$N_c^{?}\ \mathcal{P}\left(\frac{G^{ia}}{N_c}, \frac{J^i}{N_c}, \frac{I^a}{N_c}\right)$$

Two equivalent representations.

# Form of $1/N_c$ Expansion ($N_F = 3$)

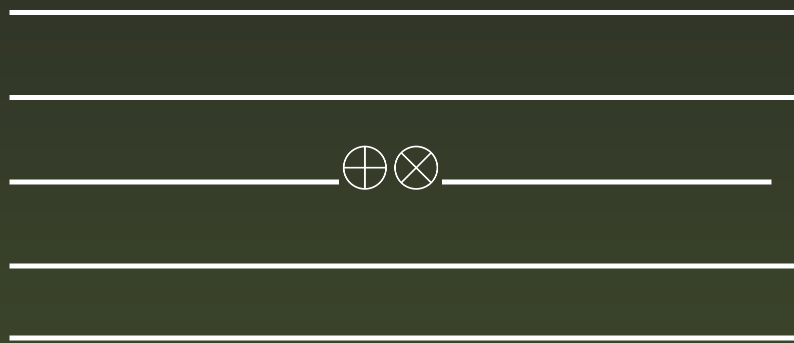
$N_F = 3$ using isospin flavor symmetry only

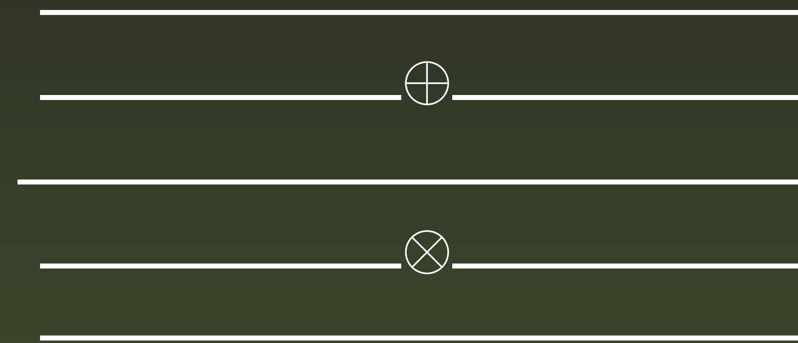
$$N_c \, \mathcal{P}\left(X_0^{ia}, \frac{J^i}{N_c}, \frac{I^a}{N_c}, \frac{S}{N_c}\right)$$

where $S$ is the strangeness. Thus $1/N_c$ constrains the form of $SU(3)$ breaking.

# $n$-body Quark Operators

- 0-body: $\mathbb{1}$
- 1-body:
$$J^i = q^\dagger \left( \frac{\sigma^i}{2} \otimes \mathbb{1} \right) q$$
$$I^a = q^\dagger \left( \mathbb{1} \otimes \frac{\tau^a}{2} \right) q$$
$$G^{ia} = q^\dagger \left( \frac{\sigma^i}{2} \otimes \frac{\tau^a}{2} \right) q$$
$$N_c = q^\dagger q$$
- 2-body: $\{J^i, G^{ja}\}$
$$\vdots$$





$$G^{ia} = \sum_{\ell=1}^{N_c} q_\ell^\dagger \left( \frac{\sigma^i}{2} \otimes \frac{\tau^a}{2} \right) q_\ell, \qquad J^i I^a = \sum_{\ell,\ell'} \left( q_\ell^\dagger \frac{\sigma^i}{2} q_\ell \right) \left( q_{\ell'}^\dagger \frac{\tau^a}{2} q_{\ell'} \right)$$

Note that commutators reduce $n$-body $\longrightarrow$ $(n-1)$-body.

# Operator Analysis

The general solution of the consistency conditions is to expand a given QCD quantity $Q$ as

$$Q = N_c^{?} \mathcal{P}\left(\frac{G^{ia}}{N_c}, \frac{J^i}{N_c}, \frac{T^a}{N_c},\right)$$

where $\mathcal{P}$ is a polynomial.
Agrees with the digrammatic analysis: each extra quark needs a gluon-exchange $\Rightarrow 1/N_c$

# $SU(3)_F$

$$\begin{array}{c}
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \\
1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \\
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1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1
\end{array}$$

$$\langle T^a \rangle \sim \begin{cases} O(1) & \text{a=1,2,3} \\ O(\sqrt{N_c}) & \text{a=4,5,6,7} \\ O(N_c) & \text{a=8} \end{cases} \qquad \langle G^{ia} \rangle \sim \begin{cases} O(N_c) & \text{a=1,2,3} \\ O(\sqrt{N_c}) & \text{a=4,5,6,7} \\ O(1) & \text{a=8} \end{cases}$$

# $SU(6)$ Operator Identities

| | |
|---|---|
| $2\ \{J^i,J^i\}+3\ \{T^a,T^a\}+12\ \{G^{ia},G^{ia}\}=5N\,(N+6)$ | $(0,0)$ |
| $d^{abc}\ \{G^{ia},G^{ib}\}+\frac{2}{3}\ \{J^i,G^{ic}\}+\frac{1}{4}\ d^{abc}\ \{T^a,T^b\}=\frac{2}{3}\,(N+3)\ T^c$ | $(0,8)$ |
| $\{T^a,G^{ia}\}=\frac{2}{3}\,(N+3)\ J^i$ | $(1,0)$ |
| $\frac{1}{3}\ \{J^k,T^c\}+d^{abc}\ \{T^a,G^{kb}\}-\epsilon^{ijk}f^{abc}\{G^{ia},G^{jb}\}=\frac{4}{3}\,(N+3)\ G^{kc}$ | $(1,8)$ |
| $-12\ \{G^{ia},G^{ia}\}+27\ \{T^a,T^a\}-32\ \{J^i,J^i\}=0$ | $(0,0)$ |
| $d^{abc}\ \{G^{ia},G^{ib}\}+\frac{9}{4}\ d^{abc}\ \{T^a,T^b\}-\frac{10}{3}\ \{J^i,G^{ic}\}=0$ | $(0,8)$ |
| $4\ \{G^{ia},G^{ib}\}=\{T^a,T^b\}\qquad(27)$ | $(0,27)$ |
| $\epsilon^{ijk}\ \{J^i,G^{jc}\}=f^{abc}\ \{T^a,G^{kb}\}$ | $(1,8)$ |
| $3\ d^{abc}\ \{T^a,G^{kb}\}=\{J^k,T^c\}-\epsilon^{ijk}f^{abc}\ \{G^{ia},G^{jb}\}$ | $(1,8)$ |
| $\epsilon^{ijk}\ \{G^{ia},G^{jb}\}=f^{acg}d^{bch}\ \{T^g,G^{kh}\}\qquad(10+\overline{10})$ | $(1,10+\overline{10})$ |
| $3\ \{G^{ia},G^{ja}\}=\{J^i,J^j\}\qquad(J=2)$ | $(2,0)$ |
| $3\ d^{abc}\ \{G^{ia},G^{jb}\}=\{J^i,G^{jc}\}\qquad(J=2)$ | $(2,8)$ |

# Operator Reduction Rule $N_F = 3$

- All operators in which two flavor indices are contracted using $\delta^{ab}$, $d^{abc}$, or $f^{abc}$ or two spin indices on $G$'s are contracted using $\delta^{ij}$ or $\epsilon^{ijk}$ can be eliminated.

# Baryon Masses

- Combined expansion in $1/N_c$ and $SU(3)$ flavor symmetry breaking
- Flavor symmetry breaking expansion extends to $3^{\mathrm{rd}}$ order in $SU(3)$ breaking
- For $N_c = 3$, only need to keep the expansion till $3$-body operators

$$M = M^1 + M^8 + M^{27} + M^{64}$$

Jenkins & Lebed

# Baryon Masses $N_F = 3$

$$M^1 = N_c \mathbb{1} + \frac{1}{N_c} J^2$$

$$M^8 = T^8 + \frac{1}{N_c} \left\{ J^i, G^{i8} \right\} + \frac{1}{N_c^2} \left\{ J^2, T^8 \right\}$$

$$M^{27} = \frac{1}{N_c} \left\{ T^8, T^8 \right\} + \frac{1}{N_c^2} \left\{ T^8, \left\{ J^i, G^{i8} \right\} \right\}$$

$$M^{64} = \frac{1}{N_c^2} \left\{ T^8, \left\{ T^8, T^8 \right\} \right\}$$

- 8 independent operators $\leftrightarrow$ 8 masses:
  $N,\ \Lambda,\ \Sigma,\ \Xi,\ \Delta,\ \Sigma^*,\ \Xi^*,\ \Omega$

# Baryon Mass Hierarchy $N_F = 3$

- Unknown coefficient multiplies each operator: order in $1/N_c$ and in $SU(3)$ flavor breaking predicted
- $\mathbf{8}$, $\mathbf{27}$, $\mathbf{64}$ operators are first, second, third order in $SU(3)$ flavor breaking parameter $\epsilon \sim m_s/\Lambda_{\mathrm{QCD}} \sim 30\%$
- Order in $1/N_c$ given by explicit factor of $1/N_c$ times leading $N_c$-dependence of operator matrix element $\langle \mathcal{O} \rangle$

# Baryon Mass Hierarchy $N_F = 3$

- Each operator contributes to unique linear combination of masses

$$\frac{J^2}{N_c}: \qquad \frac{1}{8}\left(2N + \Lambda + 3\Sigma + 2\Xi\right) - \frac{1}{10}\left(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega\right)$$

- Define dimensionless quantity

$$\frac{\sum B_i}{\sum |B_i|/2}$$

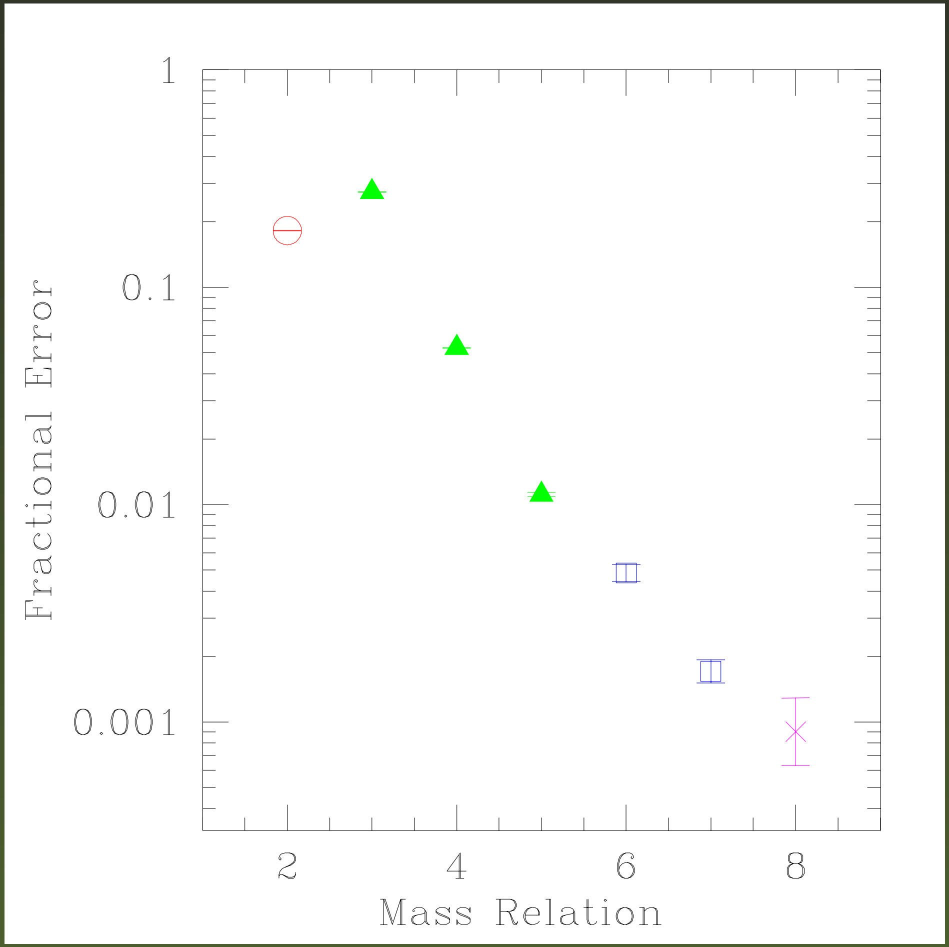
Theory: $1/N_c^2$ Expt: $(18.21 \pm 0.03)\%$

# Baryon Mass Hierarchy

| Mass Splitting | $1/N_c$ | Flavor | Expt. |
|---|---|---|---|
| $\frac{5}{8}(2N+3\Sigma+\Lambda+2\Xi)-\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$ | $N_c$ | $1$ | $*$ |
| $\frac{1}{8}(2N+3\Sigma+\Lambda+2\Xi)-\frac{1}{10}(4\Delta+3\Sigma^*+2\Xi^*+\Omega)$ | $1/N_c$ | $1$ | $18.21\pm 0.03\%$ |
| $\frac{5}{2}(6N-3\Sigma+\Lambda-4\Xi)-(2\Delta-\Xi^*-\Omega)$ | $1$ | $\epsilon$ | $20.21\pm 0.02\%$ |
| $\frac{1}{4}(N-3\Sigma+\Lambda+\Xi)$ | $1/N_c$ | $\epsilon$ | $5.94\pm 0.01\%$ |
| $\frac{1}{2}(-2N-9\Sigma+3\Lambda+8\Xi)+(2\Delta-\Xi^*-\Omega)$ | $1/N_c^2$ | $\epsilon$ | $1.11\pm 0.02\%$ |
| $\frac{5}{4}(2N-\Sigma-3\Lambda+2\Xi)-\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$ | $1/N_c$ | $\epsilon^2$ | $0.37\pm 0.01\%$ |
| $\frac{1}{2}(2N-\Sigma-3\Lambda+2\Xi)-\frac{1}{7}(4\Delta-5\Sigma^*-2\Xi^*+3\Omega)$ | $1/N_c^2$ | $\epsilon^2$ | $0.17\pm 0.02\%$ |
| $\frac{1}{4}(\Delta-3\Sigma^*+3\Xi^*-\Omega)$ | $1/N_c^2$ | $\epsilon^3$ | $0.09\pm 0.03\%$ |

(From Jenkins & Lebed)

# Baryon Mass Hierarchy




$$\frac{1}{N_c^2} : \frac{\epsilon}{N_c} : \frac{\epsilon}{N_c^2} : \frac{\epsilon}{N_c^3} : \frac{\epsilon^2}{N_c^2} : \frac{\epsilon^2}{N_c^3} : \frac{\epsilon^3}{N_c^3}$$

# Isospin Splittings

Jenkins and Lebed

Get relations that work to $0.1$ MeV accuracy. Clear evidence for the $1/N$ hierarchy. Many relations cannot be tested because the baryon masses are not well-measured.

One prediction, that the Coleman-Glashow relation

$$(p-n) - \left(\Sigma^+ - \Sigma^-\right) + \left(\Xi^0 - \Xi^-\right)$$

should work more accurately, because it is of order $\epsilon\epsilon'/N_c$ has been confirmed recently due to a more accurate measurement of the $\Xi^0$ mass.

# Baryon Axial Vector Couplings

Octet ($B$) and Decuplet ($T$) baryons have an interaction

$$
\begin{aligned}
&2D \ \mathrm{Tr}\ \bar{B} S^\mu \left\{\mathcal{A}_\mu, B\right\} + 2F \ \mathrm{Tr}\ \bar{B} S^\mu \left[\mathcal{A}_\mu, B\right] \\
&+C \ \left(\bar{T}^\mu \mathcal{A}_\mu B + \bar{B} \mathcal{A}_\mu T^\mu\right) + 2H \ \bar{T}^\mu S^\nu \mathcal{A}_\nu T_\mu
\end{aligned}
$$

Large $N_c$ predicts (to an accuracy $1/N_c^2$)

$$
F/D = 2/3, \qquad C = -2D, \qquad H = -3F \ .
$$

which agrees with experimental fit using chiral perturbation theory.

A more detailed analysis including $SU(3)$ breaking gives a good description of the data.
One result (to all orders in $SU(3)$ breaking) is that the pion coupling has the form

$$g = N_c \left( A + B \frac{S}{N_c} + \ldots \right)$$

$$\begin{aligned} g(\Sigma^* \longrightarrow \Sigma\pi) - g(\Delta \longrightarrow N\pi) &= g(\Xi^* \longrightarrow \Xi\pi) - g(\Sigma^* \longrightarrow \Sigma\pi) \\ g(\Sigma^* \longrightarrow \Sigma\pi) &= g(\Sigma^* \longrightarrow \Lambda\pi). \end{aligned}$$

# Isovector Magnetic Moments

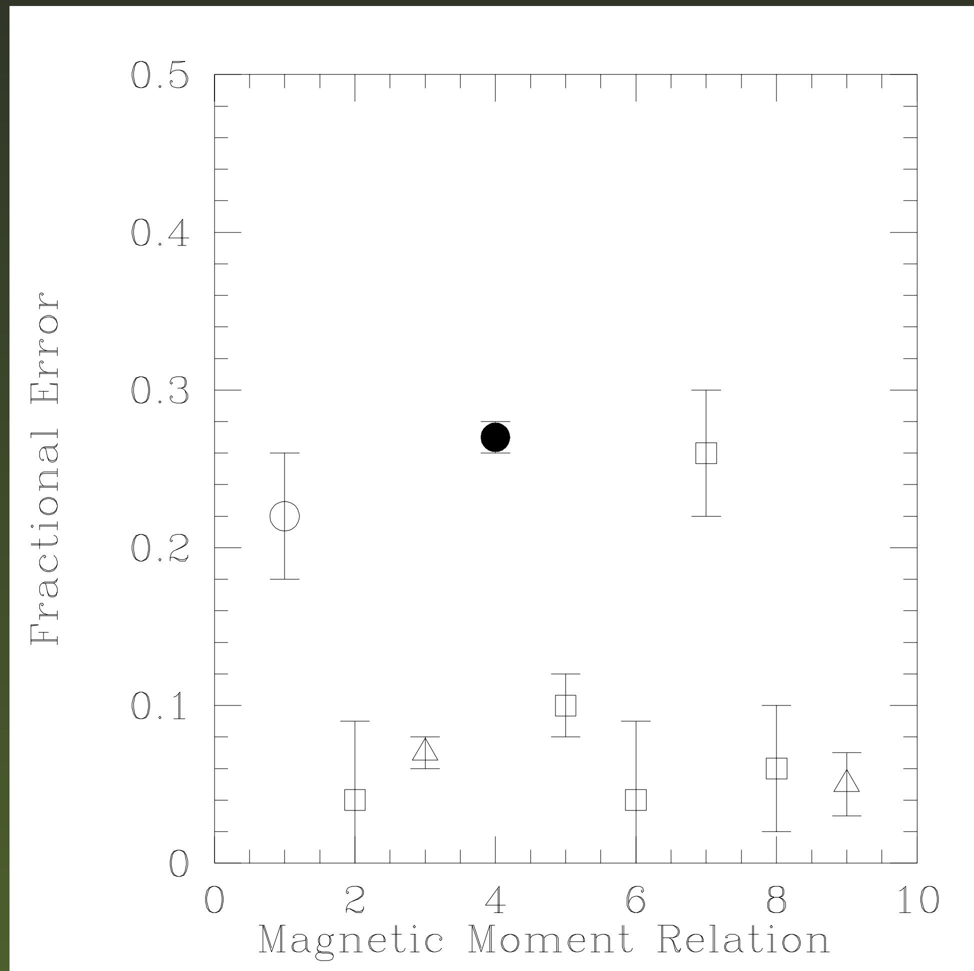
| | Isovector | $1/N_c$ | Flavor | Expt. |
|---|---|---|---|---|
| V1 | $(p-n)-3(\Xi^0-\Xi^-)=2(\Sigma^+-\Sigma^-)$ | $1/N_c$ | – | $10\pm2\%$ |
| V2 | $\Delta^{++}-\Delta^-=\frac{9}{5}(p-n)$ | $1/N_c$ | – | |
| V3 | $\Lambda\Sigma^{*0}=-\sqrt{2}\Lambda\Sigma^0$ | $1/N_c$ | – | |
| V4 | $\Sigma^{*+}-\Sigma^{*-}=\frac{3}{2}(\Sigma^+-\Sigma^-)$ | $1/N_c$ | – | |
| V5 | $\Xi^{*0}-\Xi^{*-}=-3(\Xi^0-\Xi^-)$ | $1/N_c$ | – | |
| V6 | $\sqrt{2}(\Sigma\Sigma^{*+}-\Sigma\Sigma^{*-})=(\Sigma^+-\Sigma^-)$ | $1/N_c$ | – | |
| V7 | $\Xi\Xi^{*0}-\Xi\Xi^{*-}=-2\sqrt{2}(\Xi^0-\Xi^-)$ | $1/N_c$ | – | |
| V8 | $-2\Lambda\Sigma^0=(\Sigma^+-\Sigma^-)$ | $1/N_c$ | – | $11\pm5\%$ |
| V9 | $p\Delta^++n\Delta^0=\sqrt{2}(p-n)$ | $1/N_c$ | – | $3\pm3\%$ |
| V10$_1$ | $(\Sigma^+-\Sigma^-)=(p-n)$ | 1 | – | $27\pm1\%$ |
| V10$_2$ | $(\Sigma^+-\Sigma^-)=\left(1-\frac{1}{N_c}\right)(p-n)$ | 1 | $\epsilon$ | $13\pm2\%$ |

# Isoscalar Magnetic Moments

| Isoscalar | $1/N_c$ | Flavor | Expt. |
|---|---|---|---|
| $(p+n)-3(\Xi^0+\Xi^-)=-3\Lambda+\frac{3}{2}(\Sigma^++\Sigma^-)-\frac{4}{3}\Omega^-$ | $1/N_c^2$ | – | $4\pm5\%$ |
| $\Delta^{++}+\Delta^- = 3(p+n)$ | $1/N_c^2$ | – | |
| $\frac{2}{3}(\Xi^{*0}+\Xi^{*-})=\Lambda+\frac{3}{2}(\Sigma^++\Sigma^-)-(p+n)+(\Xi^0+\Xi^-)$ | $1/N_c^2$ | – | |
| $\Sigma^{*+}+\Sigma^{*-} = \frac{3}{2}(\Sigma^++\Sigma^-)+3\Lambda$ | $1/N_c^2$ | – | |
| $\frac{3}{\sqrt{2}}(\Sigma\Sigma^{*+}+\Sigma\Sigma^{*-})=3(\Sigma^++\Sigma^-)-(\Sigma^{*+}+\Sigma^{*-})$ | $1/N_c^2$ | – | |
| $\frac{3}{\sqrt{2}}(\Xi\Xi^{*0}+\Xi\Xi^{*-})=-3(\Xi^0+\Xi^-)+(\Xi^{*0}+\Xi^{*-})$ | $1/N_c^2$ | – | |
| $5(p+n)-(\Xi^0+\Xi^-)=4(\Sigma^++\Sigma^-)$ | $1/N_c$ | – | $22\pm4\%$ |
| $(p+n)-3\Lambda=\frac{1}{2}(\Sigma^++\Sigma^-)-(\Xi^0+\Xi^-)$ | $1/N_c$ | $\epsilon$ | $7\pm1\%$ |

# Additional Relations

| Isoscalar/Isovector Relations | $1/N_c$ | Flavor | Expt. |
|---|---|---|---|
| $(\Sigma^+ + \Sigma^-) - \frac{1}{2}(\Xi^0 + \Xi^-) = \frac{1}{2}(p+n) + 3\left(\frac{1}{N_c} - \frac{2}{N_c^2}\right)(p-n)$ | $1$ | $\epsilon$ | $10 \pm 3\%$ |
| $\Delta^{++} = \frac{3}{2}(p+n) + \frac{9}{10}(p-n)$ | $1/N_c^2$ | $-$ | $21 \pm 10\%$ |




$\bigcirc = 1/N_c$, $\square = 1/N_c^2$, $\triangle = \epsilon/N_c$

# $\Delta \to N\gamma$

(Jenkins, Ji, AM)

Two helicity amplitudes, and one finds

$$\begin{aligned} \frac{A_{3/2}}{A_{1/2}} &= \sqrt{3} + \mathcal{O}\left(\frac{1}{N_c^2}\right) \\ &= 1.73(1.89 \pm 0.10) \end{aligned}$$

Equivalently,

$$\begin{aligned} \frac{E2}{M1} &= \mathcal{O}\left(\frac{1}{N_c^2}\right) \\ &= -0.025 \pm 0.005 \end{aligned}$$

# Nucleon-Nucleon Potential

D.B. Kaplan & AM

$$
\begin{aligned}
&V_{NN} = \\
&V_0^0 + V_\sigma^0 \sigma_1 \cdot \sigma_2 + V_{LS}^0 \mathbf{L} \cdot \mathbf{S} + V_T^0 S_{12} + V_Q^0 Q_{12} \\
&+(V_0^1 + V_\sigma^1 \sigma_1 \cdot \sigma_2 + V_{LS}^1 \mathbf{L} \cdot \mathbf{S} + V_T^1 S_{12} + V_Q^1 Q_{12}) \tau_1 \cdot \tau_2
\end{aligned}
$$

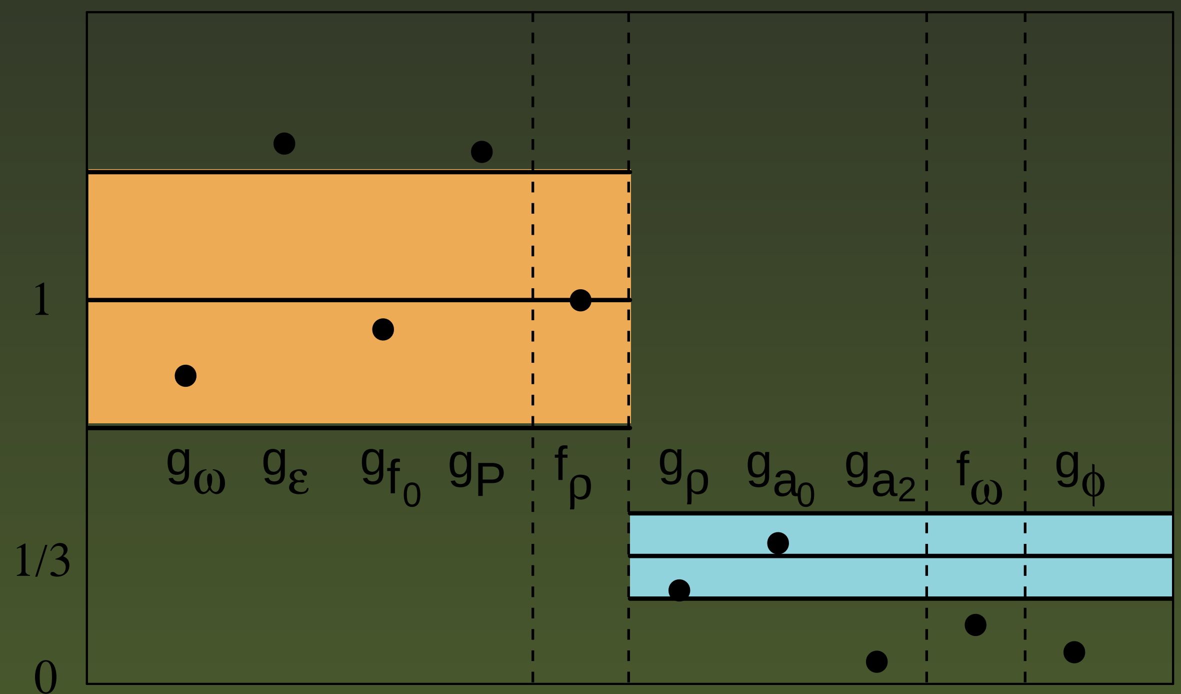
where

$$
\begin{aligned}
S_{12} &\equiv 3\sigma_1 \cdot \hat{\mathbf{r}}\ \sigma_2 \cdot \hat{\mathbf{r}} - \sigma_1 \cdot \sigma_2 \\
Q_{12} &= \frac{1}{2}\{(\sigma_1 \cdot \mathbf{L}), (\sigma_2 \cdot \mathbf{L})\}
\end{aligned}
$$

| Isospin | $V_0$ | $V_\sigma$ | $V_{LS}$ | $V_T$ | $V_Q$ |
|---|---|---|---|---|---|
| $\mathbf{1}\cdot\mathbf{1}$ | $N_c$ | $1/N_c$ | $1/N_c$ | $1/N_c$ | $1/N_c^3$ |
| $\tau_{\mathbf{1}}\cdot\tau_{\mathbf{2}}$ | $1/N_c$ | $N_c$ | $1/N_c$ | $N_c$ | $1/N_c$ |

The potential in the large $N_c$ limit has Wigner supermultiplet symmetry under which the $p\uparrow$ $p\downarrow$, $n\uparrow$, $n\downarrow$ transform as a $\mathbf{4}$ of $SU(4)$.

# Fit to parameters in the Nijmegen potential



# $I = J$ Rule

Mattis, Braaten

Mattis' $I = J$ Rule and its generalization:
Couplings are of order $N^{1-|I-J|/2}$.

e.g the $\rho$ is $I = 1$, so the dominant $\rho$ coupling is $J = 1$, i.e. magnetic moment-like ($F_2$ form factor). The $\omega$ has $I = 0$, and its dominant coupling is $J = 0$, i.e. charge-like ($F_1$).

For the $\rho$, $F_2/F_1 \sim 3$
For the $\omega$, $F_1/F_2 \sim 3$.

# Heavy Baryons

Jenkins

Form a $\overline{\mathbf{3}}$ $\Lambda_Q$ and $\Xi_Q$ and a $\mathbf{6}$, $\Sigma_Q$, $\Xi'_Q$, $\Omega_Q$ (and their spin-3/2 partners).

Heavy-quark hyperfine splittings $(\Sigma^*_Q - \Sigma_Q)$ are $150$ MeV for the $c$, and $60$ MeV for the $b$

Light-quark hyperfine splittings $(\Xi'_Q - \Xi_Q)$ are $150$ MeV.

In this case, results before the measurements.

- Can relate the pion couplings of heavy baryons to those of the $p$ up to corrections of order $1/N_c$ (rather than $1/N_c^2$).

- Obtain mass relations for heavy baryons, e.g.

$$\frac{1}{3}\left(\Sigma_Q + \Sigma_Q^*\right) = \frac{2}{3}\left(\Delta - N\right) + \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

- The $\Xi_c'$ mass was predicted to be $2580 \pm 2.1$ MeV before the measurement of $2576.5 \pm 2.3$ MeV.
- Predictions for other $c$ baryon masses, and all the $b$ baryon masses in terms of the $\Lambda_b$ mass.

# Conclusions

- $1/N_c$ expansion useful and predictive for QCD baryons, and most of the spin-flavor structure of baryons can be understood using the $1/N_c$ expansion.
- $1/N_c$ hierarchy evident in baryon masses, axial couplings and magnetic moments
- Intricate pattern of spin-flavor breaking since $1/N_c$ and $SU(3)$ breaking comparable. Restricts the form of $SU(3)$ breaking, and so is important in understanding baryon chiral perturbation theory
- Provides a unifying symmetry that connects QCD with various models such as the quark and Skyrme model.